AP Calculus AB

Review Week 1

Limits and Continuity

Advanced Placement AAP Review will be held in **room 315** and **312** on Tuesdays and Thursdays.

The week of March 23rd we will be reviewing **Limits and Continuity**.

The session will begin in room 315 with a brief review of the weekly topic.

Instruction will be from 3:00 pm to 3:15 pm

Once we have reviewed the topic you may begin practicing the questions in your review packet.

Answers will be posted in room 315 and 312 all week and will be posted on line after 3:00 pm on Friday the week of review.

If you have difficulty with a question look at the detailed answer postings BEFORE you ask your teacher for help.

Get a hint….**DON'T COPY THE ANSWER!!! THAT IS NOT HELPFUL!!**

When you have completed a question…**REFLECT!!!!** Ask yourself what skill you used to solve that problem and write that down!!

Once we have completed the weekly review, keep it to study from as we get closer to the exam.

Limits and Continuity

Brief Review

Limit – intended height (y-value) of the function.

Properties: add, subtract, divide, multiply, multiply constant and raise to any power.

Techniques to Evaluation:

- Direct Substitution – plug the x-value in…if you get a number you are done…if you get an indeterminate form….

1.) Try to factor the expression. Cancel common factors and try direct substitution again.

2.) Try tables or graphs….try plugging in a number close to the x-value to the right and the left.

3.) If you are in BC Calculus try L'Hopital's Rule or a logarithm.

One sided limits:

$$\lim_{x \to c^+} f(x) \quad is\ a\ limit\ from\ the\ RIGHT$$

$$\lim_{x \to c^-} f(x) \quad is\ a\ limit\ from\ the\ LEFT$$

Limits that approach infinity:

If it's a rational function….take the largest term on the top and bottom and simplify and then take the limit.

Remember: 1/small = BIG (infinity) 1/BIG = SMALL(zero) …..and it doesn't matter if that 1 is a 4 or a 10 or a - 3.

CONTINUITY:

1.) Function value must exist.

2.) Limit must exist.

3.) Function value must equal the limit,

Non-Calculator Active - 2008

1. $\lim_{x\to\infty}\dfrac{(2x-1)(3-x)}{(x-1)(x+3)}$ is

(A) -3 (B) -2 (C) 2 (D) 3 (E) nonexistent

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5. $\lim_{x\to 0}\dfrac{5x^4+8x^2}{3x^4-16x^2}$ is

(A) $-\dfrac{1}{2}$ (B) 0 (C) 1 (D) $\dfrac{5}{3}+1$ (E) nonexistent

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$$f(x)=\begin{cases}\dfrac{x^2-4}{x-2} & \text{if } x\neq 2\\ 1 & \text{if } x=2\end{cases}$$

6. Let f be the function defined above. Which of the following statements about f are true?

I. f has a limit at $x=2$.

II. f is continuous at $x=2$.

III. f is differentiable at $x=2$.

(A) I only

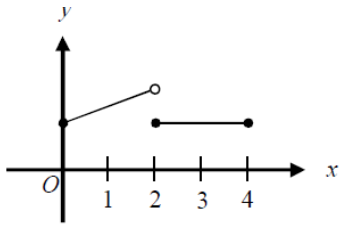
(B) II only

(C) III only

(D) I and II only

(E) I, II, and III

Calculator Active - 2008



Graph of f

77. The figure above shows the graph of a function f with domain $0 \le x \le 4$. Which of the following statements are true?

I. $\lim_{x \to 2^-} f(x)$ exists.

II. $\lim_{x \to 2^+} f(x)$ exists.

III. $\lim_{x \to 2} f(x)$ exists.

(A) I only (B) II only (C) I and II only (D) I and III only (E) I, II, and III

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89. The function f is continuous for $-2 \le x \le 2$ and $f(-2) = f(2) = 0$. If there is no c, where $-2 < c < 2$, for which $f'(c) = 0$, which of the following statements must be true?

(A) For $-2 < k < 2$, $f'(k) > 0$.

(B) For $-2 < k < 2$, $f'(k) < 0$.

(C) For $-2 < k < 2$, $f'(k)$ exists.

(D) For $-2 < k < 2$, $f'(k)$ exists, but f' is not continuous.

(E) For some k, where $-2 < k < 2$, $f'(k)$ does not exist.

Non-Calculator Active 2003

3. For $x \geq 0$, the horizontal line $y = 2$ is an asymptote for the graph of the function f. Which of the following statements must be true?

   (A) $f(0) = 2$

   (B) $f(x) \neq 2$ for all $x \geq 0$

   (C) $f(2)$ is undefined.

   (D) $\lim_{x \to 2} f(x) = \infty$

   (E) $\lim_{x \to \infty} f(x) = 2$

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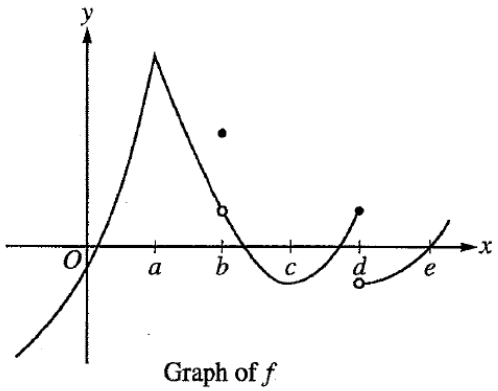
6. $\lim_{x \to \infty} \dfrac{x^3 - 2x^2 + 3x - 4}{4x^3 - 3x^2 + 2x - 1} =$

   (A) 4 (B) 1 (C) $\frac{1}{4}$ (D) 0 (E) -1



Graph of f

13. The graph of a function f is shown above. At which value of x is f continuous, but not differentiable?

   (A) a (B) b (C) c (D) d (E) e

$$f(x) = \begin{cases} x + 2 & \text{if } x \leq 3 \\ 4x - 7 & \text{if } x > 3 \end{cases}$$

20. Let f be the function given above. Which of the following statements are true about f ?

I. $\lim_{x \to 3} f(x)$ exists.

II. f is continuous at $x = 3$.

III. f is differentiable at $x = 3$.

(A) None

(B) I only

(C) II only

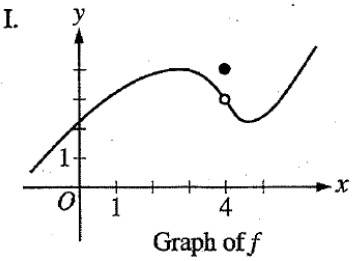
(D) I and II only

(E) I, II, and III

Calculator Active – 2003

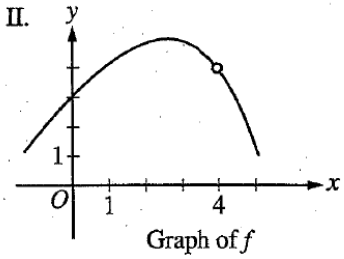
79. For which of the following does $\lim_{x \to 4} f(x)$ exist?

I.



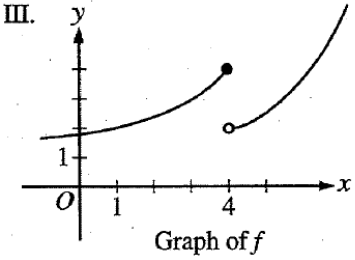
Graph of f

II.



Graph of f

III.



Graph of f

(A) I only

(B) II only

(C) III only

(D) I and II only

(E) I and III only

Free Response 2011 #6 Non-Calculator Active

6. Let f be a function defined by $f(x) = \begin{cases} 1 - 2\sin x & \text{for } x \le 0 \\ e^{-4x} & \text{for } x > 0. \end{cases}$

(a) Show that f is continuous at $x = 0$.

Free Response 2011B #2 Calculator Active

2. A 12,000-liter tank of water is filled to capacity. At time $t = 0$, water begins to drain out of the tank at a rate modeled by $r(t)$, measured in liters per hour, where r is given by the piecewise-defined function

$$r(t) = \begin{cases} \dfrac{600t}{t+3} & \text{for } 0 \le t \le 5 \\ 1000e^{-0.2t} & \text{for } t > 5 \end{cases}$$

(a) Is r continuous at $t = 5$? Show the work that leads to your answer.

Free Response 2008 #6 Non-Calculator Active

6. Let f be the function given by $f(x) = \frac{\ln x}{x}$ for all $x > 0$. The derivative of f is given by $f'(x) = \frac{1 - \ln x}{x^2}$.

(d) Find $\lim_{x \to 0^+} f(x)$.

Free Response 2003 #6 Non-Calculator Active

6. Let f be the function defined by

$$f(x) = \begin{cases} \sqrt{x+1} & \text{for } 0 \le x \le 3 \\ 5 - x & \text{for } 3 < x \le 5. \end{cases}$$

(a) Is f continuous at $x = 3$? Explain why or why not.

Free Response Practice

Given the function $f(x) = \dfrac{x^3 + 2x^2 - 3x}{3x^2 + 3x - 6}$.

(a) What are the zeros of $f(x)$?

(b) What are the vertical asymptotes of $f(x)$?

(c) The end behavior model of $f(x)$ is the function $g(x)$. What is $g(x)$?

(d) What is $\lim_{x \to \infty} f(x)$? What is $\lim_{x \to \infty} \dfrac{f(x)}{g(x)}$?